

Quantum teleportation of EPR pair by three-particle entanglement

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Abstract

Teleportation of an EPR pair using triplet in state of the Horne-Greenberger-Zeilinger form to two receivers is considered. It needs a three-particle basis for joint measurement. By contrast the one qubit teleportation the required basis is not maximally entangled. It consists of the states corresponding to the maximally entanglement of two particles only. Using outcomes of measurement both receivers can recover an unknown EPR state however one of them can not do it separately. Teleportation of the N-particle entanglement is discussed.

1 Introduction

The teleportation process, proposed by Bennett et al [1], allows to transmit an unknown state of a quantum system from a sender, traditionally named Alice, to receiver, or Bob, both are spatially separated. For teleporting of a two-state particle, or qubit, it needs an EPR pair and a usual communication channel. The large number of versions using two-particle entanglement has been considered [2]. Quantum teleportation of the photon polarized [3] and a single coherent mode of field [4] has been demonstrated in optical experiments.

As a source of EPR pair light can be used, particularly the light of the optical parametric oscillator or down conversion as in [4]. However the physical nature of the particles may be different, for instance one can choose the EPR-correlated atoms realized experimentally in [5]. Indeed, the particles of different nature are introduced in the scheme called the inter-space teleportation [6], where quantum state is transferred, for example, between the atom and light [7].

In this work we consider teleportation of an entangled pair to two distant parties Bob and Claire with the use of the triplet in the Greenberger-Horne-Zeilinger state (GHZ). Indeed the GHZ triplet has been realized experimentally [8]. The main problem is to find the three-particle projection basis for a joint measurement. By contrast the single qubit state teleportation, the maximally entangled basis does not accomplish the task. The obtained basis consist of a set of the three-particle projection operators with the maximally entanglement of two particles only. Measuring allows both receivers to recover an unknown state of EPR pair, but each of them can not do it separately. As it has been shown in ref. [9], where teleportation of a single qubit using the GHZ triplet has been considered, only one of the receivers and not both can recovered an unknown state. Our results are generalized for the teleportation of the N-particle entanglement as the EPR-nplet.

Our work is organized as follows. In section 2 the main features of the teleportation of a single qubit are given. The initial states of the entangled pair and triplet are discussed in section 3. In section 4 the basis for the joint measurement is found. The teleportation protocol and network are presented in section 5, where the results are generalized for the N- particle entanglement.

2 Teleportation

The teleportation of an unknown quantum state between two parties spatially separated, Alice and Bob, includes the following steps [1]. Let Alice has a two level system or qubit prepared in an unknown state

$$|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

where $|\alpha|^2 + |\beta|^2 = 1$. Let Alice and Bob share a maximally entangled EPR pair $|\Psi_{23}\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$, so that qubit 2 is for Alice and qubit 3 is for Bob. First, Alice performs a joint measurement of qubits 1 and 2 in

the Bell basis consisting of four projectors $\Pi_k = |\pi_k\rangle\langle\pi_k|$, $k = 1, \dots, 4$, $|\pi_1\rangle = |\Phi_{12}^+\rangle$, $|\pi_2\rangle = |\Phi_{12}^-\rangle$, $|\pi_3\rangle = |\Psi_{12}^+\rangle$, $|\pi_4\rangle = |\Psi_{12}^-\rangle$, where the Bell states are the maximally entanglement of two particles

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \quad (2)$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) \quad (3)$$

As result of the joint measurement, the density operator of the combined system $\rho = |\psi_1\rangle\langle\psi_1| \otimes |\Psi_{23}\rangle\langle\Psi_{23}|$, to be defined in the three-particle Hilbert space $H_1 \otimes H_2 \otimes H_3$, is projected into one of four Bell states. Two point are important in this procedure, first, the k -th outcome depends not on ψ_1 second, the reduced density matrix of the qubit 3 $\rho_3(k) = \text{Sp}_{12}\{\Pi_k \rho \Pi_k^\dagger\}$ and unknown state both are connected by the unitary transformation U_k

$$\rho_3(k) = U_k \tilde{\rho}_1 U_k^\dagger \quad (4)$$

where $\tilde{\rho}_1$ is the density operator of H_3 , that is the counterpart state $\rho_1 = |\psi_1\rangle\langle\psi_1|$, U_k is the set of the Pauli matrices $U_1 = \sigma_x$, $U_2 = -i\sigma_y$, $U_3 = 1$, $U_4 = \sigma_z$. Finally Alice sends the outcomes of her measurement to Bob who performs on his qubit 3 one of four unitary operations, corresponding Alice' message and has his qubit in the original state ψ_1 . Teleportation is achieved.

3 Initial states

To teleport an EPR pair it needs a maximally entanglement of three particles. From this fact let consider what initial states would be used.

The wave function of an entangled pair can be chosen as

$$|\Psi_{12}\rangle = \alpha|00\rangle + \beta|11\rangle \quad (5)$$

where $|\alpha|^2 + |\beta|^2 = 1$, or in the form of EPR-pair

$$|\Psi_{EPR}\rangle = \alpha|01\rangle + \beta|10\rangle \quad (6)$$

It is possible to point eight states where three particles are maximally entangled. They are

$$\begin{aligned} &(|000\rangle \pm |111\rangle)/\sqrt{2}, & (|001\rangle \pm |110\rangle)/\sqrt{2}, \\ &(|010\rangle \pm |101\rangle)/\sqrt{2}, & (|100\rangle \pm |011\rangle)/\sqrt{2} \end{aligned} \quad (7)$$

From the presented set of the initial states of particles without loss of generality we choose (6) and triplet in the form of GHZ

$$|\Psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \quad (8)$$

Now we shall consider the combined system prepared initially in the state

$$|\Psi\rangle = |\Psi_{EPR}\rangle \otimes |\Psi_{GHZ}\rangle \quad (9)$$

The scheme to teleport an unknown state of EPR pair of qubit 1 and 2 with the use of the GHZ triplet of qubit 3,4 and 5 is presented in fig. 1. Here three parties spatially separated, Alice and two receivers Bob and Claire share the GHZ particles 3,4 and 5. Alice sends outcomes of her joint measurement of qubits 1,2 and 3 to both receivers by classical channel. To perform the joint measurement it needs eight projection operators to form a complete set on which the initial wave function $|\Psi\rangle$ can be decomposed. The choice of such basis is the main moment in the solution of the problem.

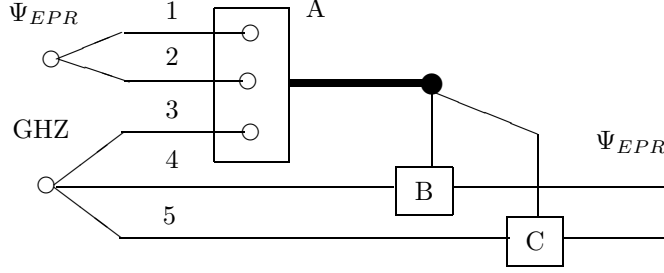


Figure 1: Teleportation of EPR pair of qubits 1 and 2 using GHZ triplet. Alice, Bob and Claire share qubits 3,4 and 5 of GHZ. Alice sends outcome of a joint measurement to Bob and Claire who recover an unknown EPR-state.

4 The projection basis

It would be possible to imagine that the set of the projection operators Π_k for joint measurement will consist of the maximally entangled states (7). Let denote this basis as $\pi_{(123)}$. However one can find that it is not true. The reason is that in series expansion of the initial wave function $|\Psi\rangle$ its projections into four vectors $(|010\rangle \pm |101\rangle)/\sqrt{2}$, $(|100\rangle \pm |011\rangle)/\sqrt{2}$ of the basis $\pi_{(123)}$ are equal to zero. It is impossible to recover unknown state of EPR pair by the such outcomes using unitary transformation. Therefore the maximally entangled tree-particle basis does not solve the task.

The basis required turns out to be composed from the states where only two particle are maximally entangled, say 1,3 or 1,2. However the pair entangled is only the necessary condition.

To consider realization of operators Π_k we introduce classification where one of tag will be number of the particles to be maximally entangled, say two or three in our case. As all complete set of vectors are connected among themselves by unitary transformation one can take an initial basis. Let the initial basis be π_{123}

$$|\pi_{123}\rangle = |ijk\rangle \quad i, j, k = 0, 1 \quad (10)$$

where each of eight elements is the state of three independent or non-correlated particles. Any element of the other basis can be presented by a linear superposition of $s \leq 8$ vectors of the set π_{123} . Further let suppose the number s be common for the given basis and we use it for classification. So it can be introduced the set $\pi_{1(23)}(s)$ consisting of the maximally entanglement of two particles 2 and 3. For $s = 2$ it has the form

$$|\pi_{1(23)}(2)\rangle = \{|i\rangle|\Phi_{23}^{\pm}\rangle; |i\rangle|\Psi_{23}^{\pm}\rangle\} \quad i = 0, 1 \quad (11)$$

where each of eight vectors, for example $|0\rangle|\Phi_{23}^{\pm}\rangle = (|000\rangle \pm |011\rangle)/\sqrt{2}$, is presented by two elements of π_{123} . For the case $s = 4$

$$|\pi_{1(23)}(4)\rangle = \{|\pi_1^{\pm}\rangle|\Phi_{23}^{\pm}\rangle; |\pi_1^{\pm}\rangle|\Psi_{23}^{\pm}\rangle\} \quad (12)$$

where pair of vectors generating a complete single-particle set looks like

$$|\pi_1^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm \exp(i\varphi)|1\rangle) \quad (13)$$

The sets presented here are complete and orthogonal, however the basis $\pi_{1(23)}(2)$ does not solve the problem. The reason is that the outcomes of the joint measurement depend on the wave function to be teleported so that the unitary transformation that Bob and Claire have to perform at their qubits will depend on an unknown state. For base $\pi_{(123)3}(4)$ the situation is similar to $\pi_{(123)}(2)$, where half of projections of Ψ into the basis states is equal to zero.

For telepoting EPR pair two sets are useful for which $s = 4$. There are $\pi_{1(23)}(4)$ ore $\pi_{(13)2}(4)$, where the particles 2,3 or 1,3 are maximally entangled. The structure of the initial state, projection basis $\pi_{1(23)}(4)$ and the total wave function are presented in fig 2.

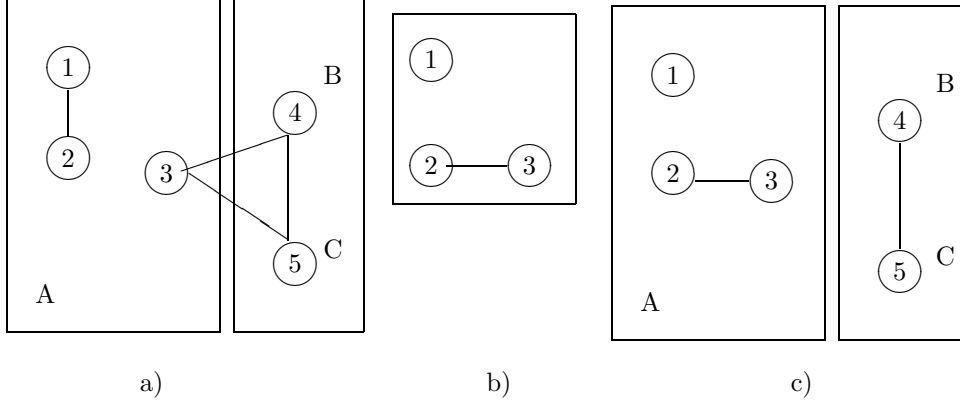


Figure 2: The entanglement structure of the states. a) The initial state, where qubits of EPR pair 1,2 and qubits 3,4,5 of GHZ are entangled. b) Projection basis $\pi_{1(23)}(4)$. c) The state after measuring.

5 Teleportation of EPR pair

Using the obtained set $\pi_{1(23)}$, where we put $\varphi = 0$, the initial wave function can be rewritten as

$$\begin{aligned}
 |\Psi\rangle &= |\pi_1^+\rangle|\Phi_{23}^+\rangle|1\rangle + |\pi_1^+\rangle|\Phi_{23}^-\rangle|2\rangle \\
 &+ |\pi_1^-\rangle|\Phi_{23}^+\rangle|3\rangle + |\pi_1^-\rangle|\Phi_{23}^-\rangle|4\rangle \\
 &+ |\pi_1^+\rangle|\Psi_{23}^+\rangle|5\rangle + |\pi_1^+\rangle|\Psi_{23}^-\rangle|6\rangle \\
 &+ |\pi_1^-\rangle|\Psi_{23}^+\rangle|7\rangle + |\pi_1^-\rangle|\Psi_{23}^-\rangle|8\rangle
 \end{aligned} \tag{14}$$

where

$$\begin{aligned}
 |1, 2\rangle &= \beta|00\rangle \pm \alpha|11\rangle \\
 |3, 4\rangle &= -(\beta|00\rangle \mp \alpha|11\rangle) \\
 |5, 6\rangle &= \beta|11\rangle \pm \alpha|00\rangle \\
 |7, 8\rangle &= -(\beta|11\rangle \mp \alpha|00\rangle)
 \end{aligned} \tag{15}$$

that for each outcome the reduced density matrix of qubit 4 and 5. It follows from equation (15) $\rho_{45}(k) = Sp_{123}\{\Pi_k|\Psi\rangle\langle\Psi|\Pi_k\}$ is connected to the density matrix of EPR pair by unitary transformation

$$\rho_{45}(k) = U_k|\tilde{\Psi}_{EPR}\rangle\langle\tilde{\Psi}_{EPR}|U_k^\dagger \quad k = 1, \dots, 8 \tag{16}$$

where $\tilde{\Psi}_{EPR}$ is the wave function of Hilbert space $H_4 \otimes H_5$ and counterpart of Ψ_{EPR} . The unitary operator from (16) has the form $U_1 = \sigma_{x4} \otimes I_5$, $U_2 = -U_3 = i\sigma_{y4} \otimes I_5$, $U_4 = -U_1$, $U_5 = I_4 \otimes \sigma_{x5}$, $U_6 = -U_7 = I_4 \otimes (-i\sigma_{y5})$, $U_8 = -U_5$. where the Pauli operators $\sigma_{\gamma j}$ $\gamma = x, y, z$ and identity I_j affect the particle $j = 4, 5$.

Teleportation of an unknown EPR state can be reached by the following protocol:

1. Alice performs the joint measurement of qubits 1,2 and 3 in basis $\pi_{1(23)}(4)$ and sends her outcomes to Bob and Claire.
2. For outcomes $k = 1 - 4$ Bob have to rotate his qubit by local operations σ_x , $i\sigma_y$, $-i\sigma_y$, $-\sigma_x$ and Claire does nothing. As result Bob and Claire has EPR pair in the state Ψ_{EPR} .
3. To recover an unknown EPR state for outcomes $k = 5 - 6$ Bob does nothing and Claire performs unitary transformation σ_x , $-i\sigma_y$, $i\sigma_y$, $-\sigma_x$ on her qubit.

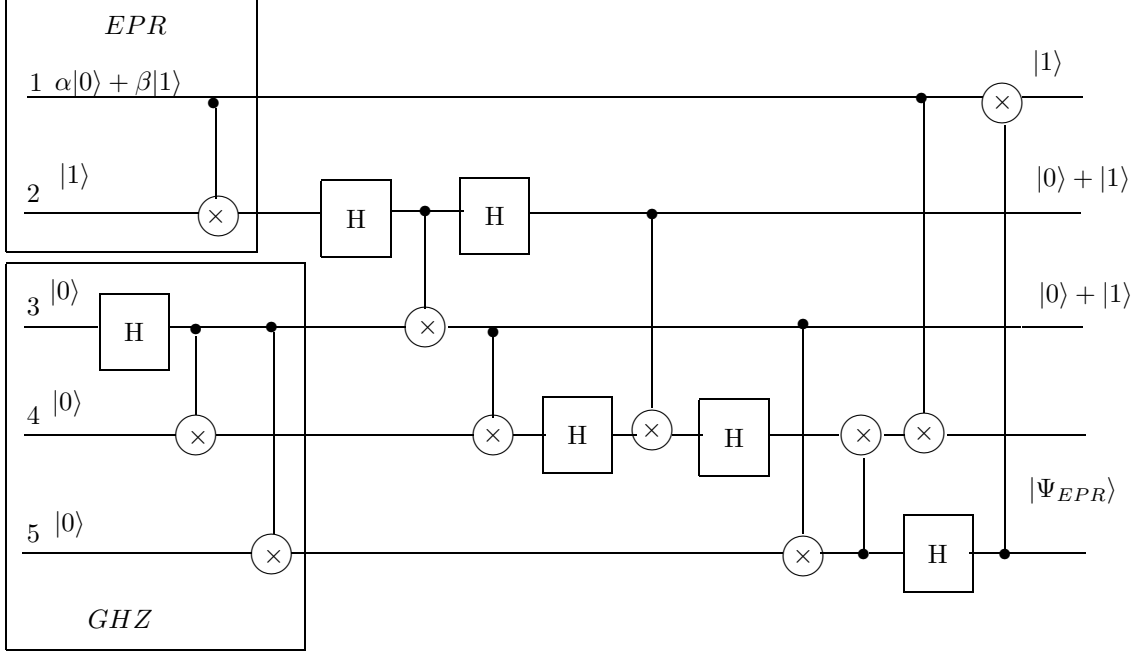


Figure 3: Network for teleportation of EPR pair

In the presented protocol in half of cases only one of receivers affects on his particle while another acts on his particle by unity operator or does nothing. This version is not unique because unknown state can be recovered by different way. For instance, the wave vector $|2\rangle$ from (15) can be obtained by two ways

$$\beta|00\rangle - \alpha|11\rangle = i\sigma_{y4} \otimes I_5 |\tilde{\Psi}_{EPR}\rangle = \sigma_{x4} \otimes \sigma_{z5} |\tilde{\Psi}_{EPR}\rangle \quad (17)$$

The expression (17) means that for outcome $k = 2$ both receivers Bob and Claire should simultaneously affect on their qubits (as in the above protocol) by unitary operations σ_{x4} and σ_{z5} (instead of $i\sigma_{y4}$ and identity operator). These differences however do not change the result. The main feature of the teleportation procedure considered here is presence of two receivers which one can not accomplish the task separately.

The network presented in fig. 3 illustrates the teleportation procedure of an EPR state. It is built similarly to the one-particle teleportation [10], and includes set of logical operations C-NOT (controlled- not) and Hadamard transformation H. In the unit *EPR* operation C-NOT C_{12} acting qubit 1,2 produces the entanglement of particles 1,2 of the form of EPR state. C_{12} flips the second qubit (target) if and only if the first (control) is logical 1. The unit *GHZ* prepares three-particle entanglement by the Hadamard transformation H acting qubit 3 ($H|0\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, $H|1\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$) and two operations c-NOT C_{34}, C_{35} . On the end of the scheme the joint state of qubits 4 and 5 being independent of others is turned out to be Ψ_{EPR} . Indeed, the above network can be used for teleporting the entangled pair of the form (5).

Consider the more general case of teleportation of the N-particle entanglement as EPR-nplet

$$|\Psi_N\rangle = \alpha|0\rangle^N + \beta|1\rangle^N \quad (18)$$

using $N + 1$ qubits in the maximally entangled state as GHZ

$$|\Psi_{(N+1)}\rangle = \frac{1}{\sqrt{2}} (|0\rangle^{N+1} + |1\rangle^{N+1}) \quad (19)$$

where $|i\rangle^N = |i\rangle \otimes \dots \otimes |i\rangle$, $i = 0, 1$. In this procedure, that includes $2N+1$ qubits, a sender Alice and N receivers share the entanglement of the form (19). The combined wave function defined in the Hilbert space $H_1 \otimes \dots \otimes H_{2N+1}$ is product $|\Psi\rangle = |\Psi_N\rangle \otimes |\Psi_{(N+1)}\rangle$.

For joint measuring of $N+1$ ($1, 2, \dots, N, N+1$) particles it needs a complete set of 2^{N+1} projectors describing states of any maximally entangled pair $M, N+1$, $M = 1, \dots, N$. For $M = N$ the required basis has the form

$$\pi_{1, \dots, N-1(N, N+1)}(s) = \{|\pi_{1, \dots, N-1}\rangle \otimes |\Phi_{N, N+1}^\pm\rangle; |\pi_{1, \dots, N-1}\rangle \otimes |\Psi_{N, N+1}^\pm\rangle\} \quad (20)$$

where the particles $N, N+1$ are entangled, $|\pi_{1, \dots, N-1}\rangle$ are the vectors from $H_1 \otimes \dots \otimes H_{N-1}$. As we note before the presence of the entangled pair is only the necessary condition for the required basis. The sufficient condition is magnitude of parameter s which one together with vectors $\pi_{1, \dots, N-1}$ can be obtained by expanding the combined wave function over set of (20). It can be written as

$$\begin{aligned} |\Psi\rangle &= \{P_{N-1}\alpha|0\rangle^N \pm Q_{N-1}\beta|1\rangle^N\}|\pi_{1, \dots, N-1}\rangle|\Phi_{N, N+1}^\pm\rangle \\ &+ \{P_{N-1}\alpha|1\rangle^N \pm Q_{N-1}\beta|0\rangle^N\}|\pi_{1, \dots, N-1}\rangle|\Psi_{N, N+1}^\pm\rangle \end{aligned} \quad (21)$$

where

$$P_{N-1} = \langle \pi_{1, \dots, N-1} | 0 \rangle^{N-1} \quad Q_{N-1} = \langle \pi_{1, \dots, N-1} | 1 \rangle^{N-1} \quad (22)$$

Process teleportation needs the following condition

$$P_{N-1} \neq 0, Q_{N-1} \neq 0 \quad (23)$$

It means that all terms of the series expansion of Ψ have to involve a linear superposition of $\alpha|i\rangle^N$ and $\beta|j\rangle^N$, $i \neq j = 0, 1$, which one can be retrieved from (18) by unitary transformation not dependent from an unknown state. The following set of vectors obeys (23)

$$|\pi_{1, \dots, N-1}\rangle = \{|\pi_1^\pm\rangle^{N-1}\} \quad (24)$$

where π_1^\pm is the one-particle set defined by (13). It can be easily established, noting that the set (24) consists of 2^{N-1} elements each of which contains two terms $|i\rangle^{N-1}$, $i = 0, 1$.

For the obtained basis defined by (20) and (24) the value s is equal to 2^N . Note, that all cases with $s < 2^N$, where there are bases inclusive more than one pair of the entangled particles (two pairs or triplet) does not accomplish the task.

As result, *for teleporting an N -particle entangled state as EPR -nplet using the $N+1$ particle entanglement it needs the set of the projection operators with one pair of the maximally entangled particles. Each element of this set has to be presented by 2^N vectors corresponding the $N+1$ -independent particle state.*

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